1 Overview of my research

I study algebraic structures coming from moduli spaces of pseudo-holomorphic curves in T^*Q and their relations to chain-level string topology of Q.

In the equivariant case, consider the symplectic field theory of the unit cotangent bundle S^*Q — an IBL-infinity structure p on the graded vector space generated by Reeb orbits — and the canonical dIBL algebra q constructed on cyclic Hochschild cochains of $H_{dR}(Q)$ using the perfect pairing $\int_Q \alpha \wedge \beta$. Consider the moduli spaces of curves in T^*Q with boundaries with marked points lying in Q and interior punctures asymptotic to Reeb orbits. Then the curves without boundary should induce an augmentation β of p, the curves without punctures a Maurer-Cartan element m in q, and the rest of the curves a morphism of the twisted IBL-infinity algebras p^β and q^m, which is a quasi-isomorphism if Q is simply connected. Moduli spaces act here as Hochschild cochains by pulling back de Rham forms at the marked points and integrating.

The Maurer-Cartan element m can be expressed up to gauge equivalence as a sum of natural contributions of trivalent ribbon graphs with interior edges decorated by a self-adjoint homotopy operator $P\colon \Omega(Q) \to \Omega(Q)$ — a Hodge propagator — and with exterior vertices decorated by elements of $H_{dR}(Q)$. The same graphs should appear as Feynman graphs for a version of Chern-Simons theory in each dimension n. Moreover, it should be possible to understand $(H_{dR}(Q), m)$ as the open sector of an open-closed string field theory.

Having discussed the equivariant case, it should be possible to apply a similar procedure in the non-equivariant case and in the case of based loops. These cases will be governed by different infinity-bialgebras. On the symplectic side, there will be chain-level symplectic homology or wrapped Floer homology, respectively. On the algebraic side, there will be different versions of Hochschild cochains and Feynman graphs with different combinatorics. Moreover, there should be natural maps between these structures on both the symplectic and the algebraic sides induced by the natural maps of the corresponding loop spaces, and they should fit into commutative diagrams.

Some literature for the paragraphs above is [6, 3, 1, 10].

Some concrete tasks I am dealing with are:

- understand the simply connected case using vanishing results and algebraic models;
- study functoriality with respect to maps of nonzero degree and formality;
- write down the theories in the non-equivariant case and in the case of based loops and study natural maps between them;
- understand to which extend is the theory determined by the tree level or curves of genus 0;

- study the relation to open-closed string field theory and apply operads and BV formalism;
- study a QFT whose BV-quantization gives the Feynman graphs in m;
- understand the transition from moduli spaces to Feynman graphs;
- extend the algebraic side to the non-simply connected case;
- extend the theory by involving a gauge group and study the relation to the Goldmann bracket and Turaev cobracket on surfaces;
- extend the theory by allowing empty boundaries;
- study analytic properties of the canonical Hodge propagator on a Riemannian manifold;
- think of how to incorporate boundary punctures asymptotic to Reeb chords.

2 Some results

My results are on the IBL-infinity chain model of equivariant string topology q^m on cyclic Hochschild cochains of $H_{dR}(Q)$ proposed in [3]. The Maurer-Cartan element m is obtained here as in [3, 7] from the formula for a pushforward of a canonical Maurer-Cartan element using a Hodge propagator P with good analytic properties. I explicitly constructed such Hodge propagator for the n-sphere and computed q^m for $n \neq 2$ in my thesis [14]. Based on this, I came up with the following:

2.1 Vanishing results

If $n \geq 3$, $H^1_{dR}(Q) = 0$, and if P is special, i.e., $P \circ P = 0$, $P \circ d \circ P = P$ (such P always exists), then the contributions of all graphs with at least one loop to the Maurer-Cartan element m are zero. Because the tree level part of m corresponds to the A-infinity homotopy transfer to $H_{dR}(Q)$ of the de Rham algebra $(\Omega(Q), d, \wedge)$, the homotopy type of q^m is determined by the homotopy type of the data of $(\Omega(Q), d, \wedge)$ and the perfect pairing $\langle\cdot,\cdot\rangle$ on $H_{dR}(Q)$ – the data of a Poincaré DGA. The vanishing results and consequences for the simply connected case are going to be published in [5]. Note that similar vanishing results were used in [2, 16].

2.2 Algebraic approach and Poincaré duality models

If $n \geq 3$ and $H^1_{dR}(Q) = 0$, one can find a Poincaré DGA $(M, d, \wedge, \langle\cdot,\cdot\rangle)$ whose pairing extends to a perfect pairing on chain level and which is weakly equivalent to $(\Omega(Q), d, \wedge, \langle\cdot,\cdot\rangle)$ — a Poincaré duality model of $\Omega(Q)$. This result comes essentially from [15]; nevertheless, I am reproving it in [13] in the language of Poincaré DGAs and consider a canonical model associated to a Hodge decomposition. The canonical dIBL algebra q_M on cyclic Hochschild cochains of M induced by the perfect pairing is then homotopy equivalent to q^m as an IBL-infinity algebra. An application of this is that Sullivan's formality of $(\Omega(Q), d, \wedge)$ implies a version of formality of q^m in the sense of IBL-infinity algebras. This result will be published in [5].

2.3 BV description of Maurer-Cartan element

The canonical dIBL algebra can be used to define a BV operator on the space of functions on the cyclic Hochschild cochains of $H_{dR}(Q)$. The Maurer-Cartan element m can then be equivalently written in terms of an action satisfying the quantum master equation. The concept of homotopy of such actions leads to the proof in [7] that m depends on P up to gauge equivalence. In fact, the BV formalism follows from the Weyl formalism of [10]. Another application is the relation of m to integrals over moduli spaces of metric ribbon graphs considered in [8], and to an open string field theory from [9]. I investigate these relations in [12].

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